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Brief Communication

On using high-order polynomial curve fits in the quasi-steady theory for square-cylinder galloping

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Abstract

Quasi-steady theory shows that the galloping response of a square cylinder exhibits a hysteresis phenomenon. The equation of motion, which was derived based on a seventh-order polynomial curve fit on the side force (C_y) versus angle of attack (α) curve, shows that the number of positive real roots corresponds to the number of stationary oscillation amplitudes. In this investigation, we use polynomials of even higher order (ninth and eleventh) to curve fit the C_y versus α curve, in an attempt to see if additional positive real roots occur, which may reveal even more flow physics. The results show that only extra negative real roots and/or complex roots are obtained when higher than seventh-order polynomial curve fits are used. Hence, the use of a seventh-order polynomial curve fit in the quasi-steady theory is shown to be sufficient in describing the flow physics which includes the prediction of the hysteresis phenomenon.

1. Introduction

Some bluff bodies immersed in a flow are susceptible to flow-induced vibrations. Transverse translational galloping has been known to affect bluff bodies with a substantial afterbody (the part downstream to flow separation positions) because its cause was traced to the interaction between the separated shear layers and the corresponding side-faces of the body. One of the most investigated bluff bodies of this category is the square cylinder (prism). The square cylinder will begin to gallop when the magnitude of the normal force (which derives from the difference in suction between its two sides) is large enough to overcome inertia and resistive (if any) forces.

A successful theory to predict the galloping response in square-cylinder flow was first proposed by Parkinson and Brooks (1961) and later in a refined version by Parkinson and Smith (1964). Known as the quasi-steady theory, it models the galloping oscillation as a linear elastic system. As the cylinder oscillates transversely with velocity $\dot{y}$ in the free stream, the instantaneous flow velocity vector is seen to approach the cylinder at an angle of incidence (α); the theory further assumes that the instantaneous transverse (or normal) fluid force acting on the oscillating cylinder is the same as the transverse force (C_y) acting on a stationary square cylinder placed at a corresponding α (and hence "quasi-steady"). It is well known that at moderate to high Reynolds number ($\mathcal{O}(10^3)$ and above), the C_y with α variation for a stationary square cylinder contains a point of inflection. Parkinson and Brooks (1961) used a fifth-order polynomial to approximate their C_y versus α data, while Parkinson and Smith (1964) used a seventh-order polynomial curve fit to capture the inflection point. The polynomial coefficients obtained from the curve fit of the C_y versus α data act as inputs

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to the equation of motion and the equation is solved for positive real roots. These roots correspond to the stationary oscillation amplitudes ($\bar{Y}$) of the galloping response at different reduced velocity (U). The quasi-steady theory (with a seventh-order polynomial) successfully predicts the hysteresis phenomenon in the galloping oscillation amplitude and this was verified experimentally in Parkinson and Smith (1964). Luo et al. (2003) concluded that the existence of the inflection point in the C_y versus α plot is a necessary condition for the existence of galloping hysteresis and it is related to the strong shear layer reattachment to the side face of the square cylinder.

The accuracy of the predictions from the quasi-steady theory is dependent on the faithful representation of the C_y versus α data by the polynomial curve fits. The work of Parkinson and Brooks (1961) and Parkinson and Smith (1964) has already demonstrated the superiority of the seventh-order polynomial fit over that of the fifth order, because only the former can capture the point of inflection in the C_y versus α relation and subsequently the hysteresis in the $\bar{Y}$ versus U relation. In view of this, one naturally questions whether a higher than seventh-order polynomial fit will lead to even better prediction and the occurrence of more positive real roots, which translates to the existence of even more stationary oscillation amplitudes. These interesting queries were first raised in our correspondence with one of the reviewers of our previous paper (Luo et al., 2003), whose input is gratefully acknowledged here; and in the present work the authors' objective is to provide the answers to the queries raised.

In this paper, we make use of three sets of C_y versus α data, namely the experimental data from Parkinson and Smith (1964), and Luo and Bearman (1990) for high Reynolds number (Re) flows, and our recent numerical data from Luo et al. (2003) for moderate Re flow. Next, seventh, ninth and eleventh-order polynomial curve fits are used to approximate all the C_y versus α data and the respective polynomial coefficients are extracted. The equation of motion is solved in order to determine the number of positive real roots (or stationary oscillation amplitudes) and whether extra roots come along when higher-order polynomial curve fits are used.

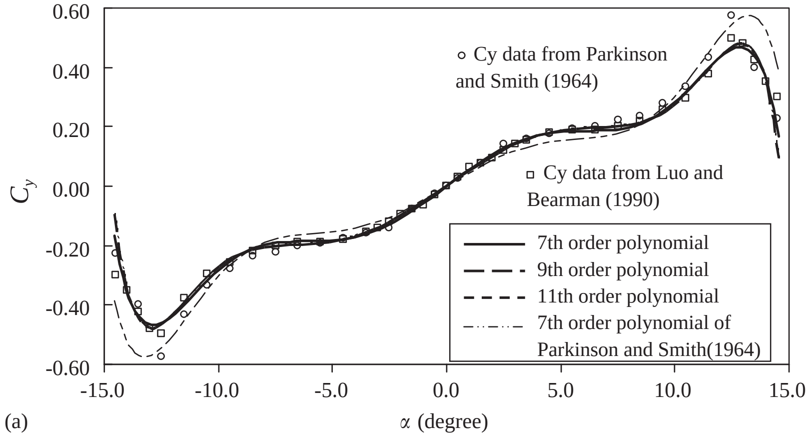
2. Polynomial curve fit and equation of motion

The force coefficient C_y, measured on a stationary square cylinder at different angle of incidence α, is usually expressed as a polynomial function of α. In the quasi-steady theory analysis, for convenience the variable is expressed as $\dot{y}/U$ ($=\tan\alpha$). The C_y versus α data from Parkinson and Smith (1964) and Luo and Bearman (1990) are re-plotted in Fig. 1(a) for a range of $\alpha = -15^\circ$ to 15° (or $\tan\alpha = -0.2679$ to 0.2679), while similar data from Luo et al. (2003) are presented in Fig. 1(b) for an α range of -10° to 10° (or $\tan\alpha = -0.1763$ to 0.1763). The data collected from the first two studies showed fairly good agreement as both were performed experimentally at similar high Re flows of $\mathcal{O}(10^4)$ (and henceforth known as high Re data). The C_y versus α data from Luo et al. (2003) was obtained numerically at a lower, Reynolds number, Re $= 1000$, (and is henceforth referred to as moderate Re data). Nevertheless, all three sets of data show a similar trend and the characteristic inflection point is present in all of them.

Next, by using Microsoft Excel (with an add-on function known as XlXtrFun$^{\text{TM}}$), seventh, ninth and eleventh-order polynomials are fitted onto the high Re data of Luo and Bearman (1990) (in the range of $-14.5^\circ \leqslant \alpha \leqslant 14.5^\circ$) and the moderate Re data of Luo et al. (2003) (in the range of $-8^\circ \leqslant \alpha \leqslant 8^\circ$) and they are also shown in Fig 1(a) and 1(b), respectively. Both figures show that there is no significant discrepancy among the fitting of the three polynomials. The important curvature changes in the C_y versus α variation are captured by all the three polynomial curve fits. In Fig 1(a), the seventh-order polynomial curve fit from Parkinson and Smith (1964) is also included. For both the high and moderate Re flows, the polynomial curve fit is given in the form

$$C_y = A\left(\frac{\dot{y}}{U}\right) + B\left(\frac{\dot{y}}{U}\right)^3 + C\left(\frac{\dot{y}}{U}\right)^5 + D\left(\frac{\dot{y}}{U}\right)^7 + E\left(\frac{\dot{y}}{U}\right)^9 + F\left(\frac{\dot{y}}{U}\right)^{11}. \quad (1)$$

We use the symbols A to F to represent the coefficients of the lowest to the highest order ($\dot{y}/U$) term in the polynomial. It is obvious that for polynomial curve fits that are of lower order than Eq. (1), the corresponding higher-order term coefficient(s) is (are) zero; (that is, $F = 0$ for a ninth-order polynomial and both E and $F = 0$ for a seventh-order polynomial). Note that we use positive signs for all the coefficients and this differs from the convention adopted in Parkinson and Smith (1964), where alternating signs were used, and where A to D themselves are positive numbers. We have adopted this new convention because we noted that the coefficients for the ninth and eleventh-order polynomial curve fits may not be positive if we adopt Parkinson and Smiths' alternate sign convention. Hence, for clarity, we include the signs of the polynomial coefficients below and the correct equation can be obtained by substituting the required coefficients, together with its sign, into Eq. (1). The coefficients of the polynomial curve fits are as shown in Table 1.



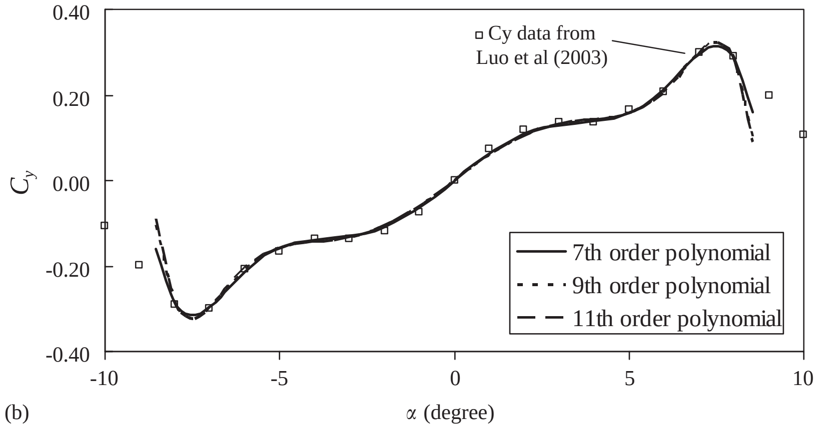


Fig. 1. (a) The C_y versus α data from Parkinson and Smith (1964) (○) and from Luo and Bearman (1990) (□). Seventh, ninth and eleventh-order polynomial curve fits are applied to the data of Luo and Bearman (1990) at high Re. Also shown is the seventh-order polynomial curve fit by Parkinson and Smith (1964). (b) The C_y versus α data from Luo et al. (2003) (□). Seventh, ninth and eleventh order polynomial curve fits are applied to these data for moderate Re flow.

Table 1
Coefficients of polynomial curve fits

	A	B	C	D	E	F
High Re						
7th order	3.303	−208.71	7042.7	−67,104		
9th order	3.037	−140.46	2781.6	27,992	−696,370	
11th order	3.043	−142.56	2988.27	20,008	−563,885	−790,856
Moderate Re						
7th order	3.808	−640.38	63,202	−1,784,690		
9th order	3.595	−483.84	32,785	363,215	−49,727,434	
11th order	3.555	−436.51	22,876	1,033,173	−63,301,669	−44,800,000

To illustrate the goodness of fit among the three polynomials and at the two different Reynolds numbers, the corresponding correlation coefficients and sums of squared error were also computed and tabulated in Table 2. The trends of these data demonstrate (as expected) that the correlation coefficient increases and the sum of squared error decreases when the order of the polynomial is increased. However, it is also clear that the improvement obtained by using a polynomial higher than seventh-order is insignificant. For example, the correlation coefficients for all three

Table 2
Correlation coefficient and the sum of squared error for the three polynomials and at the two Reynolds number

	7th order	9th order	11th order
Moderate Re			
Sum of squared error	1.554788×10^{-3}	1.444595×10^{-3}	1.435142×10^{-3}
Correlation	0.992511	0.998775	0.998783
High Re			
Sum of squared error	0.0115	0.0043	0.0014
Correlation	0.9982	0.9993	0.9997

polynomials are 0.99+. Improvement only takes places from the third decimal place (less than 1%). This comparison therefore further demonstrates that the seventh-order polynomial provides a good enough fit for the data in the α range stated.

From quasi-steady theory, the equation of motion will have additional terms when higher-order polynomial curve fits are used for approximating the C_y versus α relation. Uninitiated readers can consult Parkinson and Smith (1964) on the details of the derivation of these additional terms. The equations of motion based on the respective polynomial curve fits are given as follows:

7th order polynomial

$$\frac{d\bar{Y}^2}{d\tau} = nA\left\{\left(U - \frac{2\beta}{nA}\right)\bar{Y}^2 + \frac{3}{4}\left(\frac{B}{AU}\right)\bar{Y}^4 + \frac{5}{8}\left(\frac{C}{AU^3}\right)\bar{Y}^6 + \frac{35}{64}\left(\frac{D}{AU^5}\right)\bar{Y}^8\right\}; \tag{2}$$

9th order polynomial

$$\frac{d\bar{Y}^2}{d\tau} = nA\left\{\left(U - \frac{2\beta}{nA}\right)\bar{Y}^2 + \frac{3}{4}\left(\frac{B}{AU}\right)\bar{Y}^4 + \frac{5}{8}\left(\frac{C}{AU^3}\right)\bar{Y}^6 + \frac{35}{64}\left(\frac{D}{AU^5}\right)\bar{Y}^8 + \frac{315}{640}\left(\frac{E}{AU^7}\right)\bar{Y}^{10}\right\}; \tag{3}$$

11th order polynomial

$$\frac{d\bar{Y}^2}{d\tau} = nA\left\{\left(U - \frac{2\beta}{nA}\right)\bar{Y}^2 + \frac{3}{4}\left(\frac{B}{AU}\right)\bar{Y}^4 + \frac{5}{8}\left(\frac{C}{AU^3}\right)\bar{Y}^6 + \frac{35}{64}\left(\frac{D}{AU^5}\right)\bar{Y}^8 + \frac{315}{640}\left(\frac{E}{AU^7}\right)\bar{Y}^{10} + \frac{3465}{7680}\left(\frac{F}{AU^9}\right)\bar{Y}^{12}\right\}. \tag{4}$$

In the above equations, $\bar{Y}$ is the dimensionless amplitude of oscillation, τ is the dimensionless time, the parameter n is half the ratio of displaced air mass to cylinder mass and β is the system damping coefficient. If we define $R = \bar{Y}^2$ in accordance with Parkinson and Smith (1964), we can rewrite the above three equations in a simplified form,

$$\frac{dR}{d\tau} = aR + bR^2 + cR^3 + dR^4 + eR^5 + fR^6 = F(R). \tag{5}$$

The coefficients a to f refer to the corresponding collected coefficients found in Eqs. (2–4). As before, the coefficients f and e will be zero for polynomials with order lower than 11 and 9, respectively. The stationary oscillation amplitudes (or positive real roots) of the galloping response correspond to the condition when $dR/d\tau = 0$. Parkinson and Smith (1964) solved the equation of motion for a seventh-order curve fit and found that besides the root $R = 0$, the equation can either have one or three positive real root(s). They further classified these roots into stable and unstable roots depending on the slope of the $F(R)$ curve.

At the onset, by looking at Eq. (5), it seems that more positive real roots will be available if a higher-order polynomial is curve-fitted onto the data. However, further investigation suggests otherwise. In this paper, Eq. (5) that corresponds to the seventh, ninth and eleventh-order polynomial curve fit is solved for its positive real roots (stationary oscillation amplitude) for the high Re case only. This is shown in Fig. 2 as a universal oscillation amplitude $(nA/2\beta)\bar{Y}$ versus reduced velocity $(nA/2\beta)U$ plot. The figure shows that the hysteresis phenomenon in galloping oscillation is reflected in all three curves. At a reduced velocity just before the hysteresis region, one positive real root (stable root) is present. In the hysteresis region, three positive real roots (two stable and one unstable root) are present, and beyond the hysteresis region, one positive real root (stable root) is present. There are no extra positive real roots when a higher order polynomial curve fit is employed. Comparison of the three sets of the present data (arising from the three different polynomial fits) suggests that they are quite close to each other. On the other hand, comparison of the present data with those of Parkinson and Smith (1964) showed some disagreement. This is due to the use of different polynomial coefficients in the approximation of the C_y versus α relation shown in Fig. 1(a). This disagreement was also noted by Norberg (1993) when he used a different seventh-order polynomial curve to approximate the C_y versus α relation at a different Re.

Since the use of higher-order polynomial curve fits does not result in the emergence of more positive real roots, we can conclude that negative R roots and pairs of complex conjugate roots appear after solving Eq. (5). These roots are discarded because $R = \bar{Y}^2$. We are able to know the number of positive or negative roots in a polynomial by counting the number of sign changes in the polynomial, based on Descartes' Rule of Sign (Henrici, 1988). To paraphrase the Descartes' Rule of Sign,

> "The number of positive roots of a polynomial, $F(R)$, with real coefficients is equal to the number of 'changes of sign' in the list of coefficients, or is less than this number by a multiple of 2. And the number of negative roots of the same polynomial is equal to the number of sign changes found in $F(-R)$ or is less by a multiple of 2".

With this rule in mind, we look at the Eq. (5) that corresponds to an eleventh-order polynomial curve fit as an example,

$$\begin{aligned} &F(R) = aR - bR^2 + cR^3 + dR^4 - eR^5 - fR^6 = 0 \\ &\text{(after noting the signs of coefficients } A \text{ to } F \text{ in Table 1)} \\ &a - bR + cR^2 + dR^3 - eR^4 - fR^5 = 0 \\ &\text{(where } R = 0 \text{ is one of the roots).} \end{aligned} \tag{6}$$

There are three sign changes (i.e. $+a$ to $-b$, $-b$ to $+c$ and lastly $+d$ to $-e$) and hence, by Descartes' Rule of Sign, there should be either 3 positive roots or 1 positive root (the number of changes of sign, less a multiple of 2). These two possible combinations coincide exactly with the curve shown in Fig. 2 when a hysteresis region is present. When a single positive root is present, this will coincide with the regions corresponding to velocity that are either lower or higher than the hysteresis region. Within the hysteresis region, three positive roots are present. To obtain the number of negative roots, Eq. (6) is rewritten as $F(-R)$,

$$F(-R) = a + bR + cR^2 - dR^3 - eR^4 + fR^5 = 0. \tag{7}$$

There are two sign changes and hence there are either two negative roots or none. However from the definition of $R = \bar{Y}^2$, both complex and negative roots are neglected in the analysis. Since $F(R)$, for an eleventh-order curve fit, is a

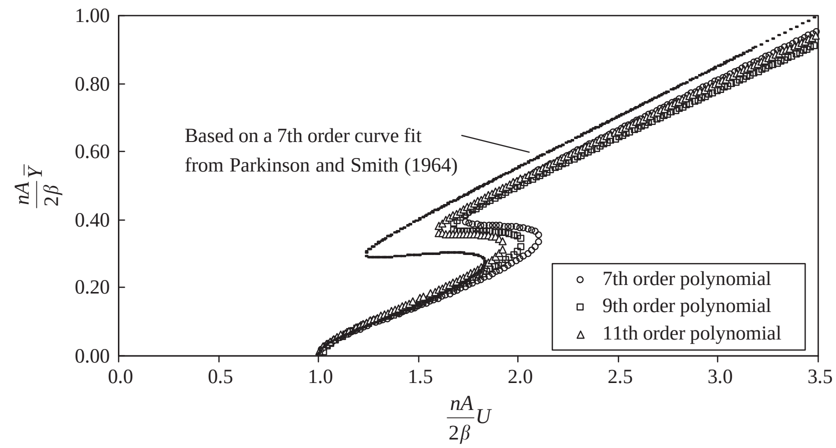


Fig. 2. A hysteresis region is captured in the universal amplitude–velocity characteristic when polynomials of order seven or higher are used to approximate the C_y versus α relation.

6th order polynomial, we expect 6 roots with the following four combinations: $R = 0$, 2 negative roots, 1 positive root, and a pair of complex roots, $\{R = 0$, 2 negative roots, 3 positive roots, and no complex roots$\}$, $\{R = 0$, no negative root, 1 positive root and 2 pairs of complex roots$\}$ and $\{R = 0$, no negative root, 3 positive roots, and a pair of complex roots$\}$.

Descartes' Rule of Sign will yield similar results when applied to Eq. (5) for a ninth-order polynomial curve fit. The number of positive real roots will remain unchanged at either one or three, while the extra roots will either be negative real roots or complex conjugate pairs. The earlier doubt on whether a higher-order polynomial fit will result in more oscillation amplitudes and multi-branch hysteresis is thus shown to be an unnecessary concern.

3. Conclusion

In the present work, we have shown that the use of higher (than the commonly used seventh) order polynomials to approximate the C_y versus α relation neither results in a significantly better fit nor in any additional positive real roots from its equation of motion. As a result of the latter, we do not expect to see any additional stationary oscillation amplitudes in the square cylinder galloping response. Although the equation of motion derived from quasi-steady theory contains higher-order terms, the extra roots turn out to be either negative or complex and hence they can be discarded. By using Descartes' Rule of Sign, we showed that the changes in sign of the seventh, ninth and eleventh-order polynomials are such that they all point to the existence of either one or three positive real root(s). In Luo et al. (2003), we demonstrated that in order to capture the hysteresis phenomenon in the $\bar{Y}$ versus U relation for a square cylinder, the use of a seventh-order polynomial in representing the C_y versus α relation in Parkinson's quasi-steady theory is necessary. In this note, we further demonstrate that a seventh-order polynomial is also sufficient. The use of a polynomial with an order higher than seven neither results in a significantly better fit nor in additional oscillation amplitudes.

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